LABORATORY INVESTIGATIONS

Design of Computer-Generated Hologram With Ring Focus for Nonmechanical Corneal Trephination With Er:YAG Laser in Penetrating Keratoplasty

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Purpose: To calculate a beam-shaping optical element for homogeneous intensity distribution within a focal ring to be used in nonmechanical trephination with the Er:YAG laser in penetrating keratoplasty instead of a spot guiding device.

Methods: The phase distribution behind a holographic optical element (HOE) $k\Psi(\mathbf{u})$ can be described by the addition of the hologram phase $\Phi_H(\mathbf{u})$ to the beam phase $\Phi_E(\mathbf{u})$: $k\Psi(\mathbf{u}) = \Phi_H(\mathbf{u}) + \Phi_E(\mathbf{u})$, $k = 2\pi/\lambda$, where **u** denotes the coordinates inside the hologram aperture, k an integer, and λ the laser wavelength. To avoid discontinuous wavefronts leading to speckle noise, a smooth phase function is necessary. After transforming the hologram aperture coordinates into the focal plane x in a focal distance f, Ψ can be retrieved from the slope equation: $\nabla\Psi(\mathbf{u}) = x(\mathbf{u}) - \mathbf{u}/f$.

Results: Creating a ring focus can be reduced to an essentially one-dimensional problem by separation of variables due to the symmetry condition. We calculated a computer-generated eight-level phase-only HOE with 4096 × 4096 pixels from a Gaussian-distributed 2.94 Er:YAG laser spot with a beam diameter of 10 mm and a focal distance of 100 mm. Thereby, a ring focus with an inner/outer radius of 7/8 mm can be created. To avoid Poisson's spot, the symmetry of the problem was broken by circular modulation of the phase leading to a spiral-like structure. The calculated efficiency of the HOE relating the energy within the ring to the total energy was 91%.

Conclusion: With an HOE it is possible to redistribute the energy along the desired focal ring. The HOE design can be adapted to the intensity distribution of the impinging laser beam with its characteristic aperture shape. A circular homogeneous corneal trephination depth is possible, because the energy fluctuation from pulse to pulse does not locally affect the ablation process. A ring focus for the Er:YAG laser has the potential to render superfluous a manual beam control via micromanipulator and to allow a more rapid and more regular corneal trephination along aperture masks. **Jpn J Ophthalmol 1999;43:453–457** © 1999 Japanese Ophthalmological Society

Key Words: Holographic optical element, penetrating keratoplasty, solid state laser.

Introduction

Nonmechanical trephination with the excimer laser in penetrating keratoplasty has been in clinical use in our department since 1989, and has been developed into a standard method.[1–3] More than 450 patients have been treated successfully with this method. The major advantages of this atraumatic method of trephination are homogeneous cut angles and surfaces, a higher degree of regularity, the use of orientation teeth for positioning of the first interrupted sutures and an arbitrary shape of the configu-

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ration of trephination. The only drawbacks of the excimer laser keratoplasty are the difficult handling of the laser and the relatively high acquisition and service costs. Looking for an inexpensive and efficient alternative laser for nonmechanical trephination, the solid-state Er:YAG laser (wavelength: 2940 nm) has promising properties.

In the clinic, a circular round excimer laser trephination with eight orientation teeth is used, and the laser beam is moved manually with a joystick along the edges of metal masks under visual control by an operating microscope. In eccentrically located corneal ulcers or peripheral corneal ectatic diseases, an elliptical trephination outline is possible. Circular and elliptical trephination outlines can be realized with a holographic optical element in combination with a spherocylindrical telescope. Thus, a ring focus can be created with a beam-shaping element to be used instead of a computer-assisted or manual beam control of a single laser spot.

Materials and Methods

For the design of beam-shaping holograms, the geometry of Figure 1 has to be considered. Coordinates inside the hologram are denoted as $\mathbf{u} = (u, v)$. An incoming laser beam, characterized by its intensity and phase distribution $IE(\mathbf{u})$ and $\Phi E(\mathbf{u})$, respectively, impinges on a phase-only hologram. For the sake of mathematical convenience, we write the resulting phase distribution behind the element as $k\Upsilon(\mathbf{u})$ with Υ being the eikonal.[4] Thus, the effect of the hologram can be described by the addition of the hologram phase $\Phi_H(\mathbf{u})$ and the laser beam phase $\Phi_E(\mathbf{u})$ (Eq. 1):

$$k\Psi(\mathbf{u}) = \Phi_H(\mathbf{u}) + \Phi_E(\mathbf{u}). \qquad (1)$$

In the reconstruction plane F, labeled by the coordinates $\mathbf{x} = (x, y)$ at a distance f behind the hologram, we want to obtain a specified intensity distribution $IF(\mathbf{x})$. To avoid discontinuous wavefronts leading to speckle noise, a smooth phase function has to be created. Hence, there must be a smooth coordinate transform $\mathbf{x}(\mathbf{u})$ describing the path of light rays passing through the hologram aperture at $\mathbf{u}$ and the focal (or reconstruction) plane at $\mathbf{x}$. After a coordinate transform $\mathbf{x}(\mathbf{u})$ has been defined, the eikonal Υ can be retrieved from the slope equation (Eq. 2):

$$\nabla\Psi(\mathbf{u}) = \frac{\mathbf{x}(\mathbf{u}) - \mathbf{u}}{f} \qquad (2)$$

which follows immediately from the geometry shown in Figure 1B in paraxial approximation. Equation 2

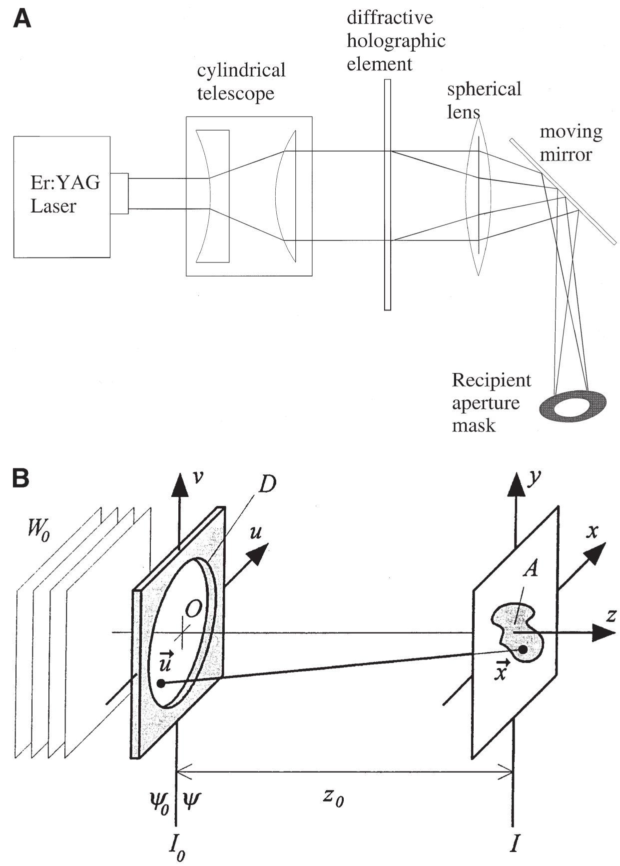

Figure 1. (**A**) Principle of nonmechanical trephination of cornea with ring focus in penetrating keratoplasty. (**B**) Reconstruction geometry: incident wave on aperture plane E is to be focused into pregiven intensity pattern at focal plane.

is an approximate ray-tracing representation that has been widely used in geometric and far-field optics.[4,5] The construction of appropriate transforms $\mathbf{x}(\mathbf{u})$ is based on the following two requirements: first, to avoid absorption losses, the total energy must be conserved (Eq. 3):

$$E = \int_E I_E(\mathbf{u})d^2\mathbf{u} = \int_F I_F(\mathbf{x})d^2\mathbf{x}. \qquad (3)$$

Second, the directive $\mathbf{x}(\mathbf{u})$ should redistribute the intensity $I_E(\mathbf{u})$ of the aperture plane into the desired reconstruction distribution $I_F(\mathbf{x})$, ie, the condition (Eq. 4):

$$I_E(\mathbf{u})d^2\mathbf{u} = I_F(\mathbf{x})d^2\mathbf{x} \qquad (4)$$

has to be fulfilled, where $d^2\mathbf{x}$ is the image of the area element $d^2\mathbf{u}$. As it is known from elementary calculus, area elements transform with the determinant of Jacobi matrix. This requirement can also be formulated as (Eq. 5):

$$I_F[\mathbf{x}(\mathbf{u})]\det\frac{\partial \mathbf{x}}{\partial \mathbf{u}} = I_E(\mathbf{u}). \tag{5}$$

In our application, a homogeneous intensity profile with a circular pupil $I_F(\mathbf{x})$ has to be transformed to a ring profile $I_E(\mathbf{u})$. This task can be reduced to a one-dimensional problem. Due to the symmetrical condition of the ring, the determinants in Eq. 5 can be integrated either analytically or numerically, whereas the design of a hologram in general requires a numerical integration.

Results

Calculation of a Ring Focus

The phase function of a ring focus with a circular aperture (radius r_0) and homogeneous intensity distribution can be calculated analytically. The description of the problem can be made with a transform to polar coordinates (r,β) in the hologram plane and (ρ,Θ) in the focal plane. The geometric center of the ring with an outer radius r_1 and an inner radius r_2 is localized on the z-axis. The intensity within the ring is given by redistributing the total beam energy $I_E = \pi r^2_0 I_E$ homogeneously into the ring structure at the reconstruction plane $A = \pi(r^2_1 - r^2_2)$ (Eq. 6):

$$I_E(\rho) = \frac{I_E}{A} = \frac{r_0^2}{r_1^2 - r_2^2} I_E \text{ for } r_2 < \rho < r_1. \tag{6}$$

The transform of Eq. 5 into polar coordinates reads (Eq. 7):

$$I_F(\rho(r))\rho d\rho = I_E(r) r dr. \tag{7}$$

The coordinate transform can be formulated by integration of Eq. 7 (Eq. 8):

$$\rho(r) = r_2\sqrt{1 + \left(\frac{r_1^2}{r_2^2} - 1\right)\frac{r^2}{r_0^2}} \text{ for } 0 < r < r_0. \tag{8}$$

The defocus is given by integration of Eq. 2 (Eq. 9):

$$\Psi(r) = \frac{r_2^2 r_0}{2f\sqrt{r_1^2 - r_2^2}} q\left(\frac{r\sqrt{r_1^2 - r_2^2}}{r_2 r_0}\right)$$

with

$$q(t) = t\sqrt{t^2 + 1} + \ln(\sqrt{t^2 + 1}). \tag{9}$$

Azimuthal Phase Modulation

With a radially symmetrical intensity pattern such as a circular or a ring focus, a marked reconstruction error is possible because of the -1 diffraction order. Along the z-axis an intensity maximum occurs. To avoid Poisson's spot as a consequence of a constructive superposition of divergent waves from all azimuthal directions Θ, the symmetry of the problem is broken by circular modulation of the phase (Eq. 10):

$$\Phi_{E,j}(r,\Theta) = \Phi_E(r,\Theta) + j\Theta \; j \in N. \tag{10}$$

The phase increment of $2\pi j$ for each cycle leads to a spiral-like structure in the binary hologram. The phase of the divergent wave from Θ has a phase shift of π to the divergent wave from the direction $\Theta + 2\pi/j$. The intensity equals zero in a circular area centered to the z-axis. The diameter of this central region depends on the degree of azimuthal modulation.

Ring Focus for Corneal Trephination

We demonstrate the calculation of a computer-generated hologram with a ring focus for an Er:YAG laser with homogeneous intensity distribution. Figure 2A shows the phase function with binary quantization of a ring focus with an aperture stop radius ($r_0 = 5$ mm) in the hologram plane. The inner diameter of the ring focus is $2r_2 = 7$ mm and the outer diameter is $2r_1 = 8$ mm at a focal distance $f = 100$ mm. The numerical simulation based on paraxial ray-tracing techniques reveals an intensity distribution in the focal plane shown in Figure 2B. The relatively high energy density at the z-axis (Poisson's spot) may effect an undesirable ablation of corneal tissue in the corneal center. With a circular fourth-order modulation of the hologram phase specified in Figure 2C, the Poisson spot at the z-axis disappears (Figure 2D). With a circular modulation, the intensity of the reconstruction error becomes smaller and a circular intensity-free area centered to the z-axis with a diameter 0.55 mm appears. The energy density at the small ring structures lies beyond the ablation threshold of corneal tissue. The theoretical efficiency of the optical element defined as the ratio of the energy within the ring to the total energy is about 91%.

In comparison to the implementation of a holographic optical element for ultraviolet lasers, the fabrication process using laser lithography and reactive ion etching is not critical. With the wavelength of the Er:YAG-laser in the mid-infrared (2.94 μm), the structures are relatively large (>4 μm).

With a spherocylindrical telescope, the fabrication of one HOE is sufficient for creating different trephination diameters or ellipses.

Discussion

A holographic optical element with a ring focus gives the possibility to use the extensive energy of

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the Er:YAG solid state laser (100–800 mJ per pulse) for an efficient and fast trephination of the cornea. Since ablation along the edges of masks placed on donor and recipient cornea allows for sharp cut edges parallel to the optical axis, the radial intensity profile in the focal plane is not required to be exactly rectangular.

The hologram function is fully determined in amplitude and phase by the application. However, the phase of the ring pattern to be reconstructed is of no concern for our clinical application, but this additional degree of freedom can be used to optimize the hologram function in terms of diffraction efficiency or simplicity. Several studies have been undertaken to develop algorithms for efficient and precise calculation of hologram functions, including Fourier algorithms and nonlinear optimization.[6–8] With the increasing speed of new computer generations, more and more numerical optimization methods were introduced and implemented to substitute analytical methods. In our application, the hologram is based on a periodic function and corresponds to a discrete Fourier spectrum.[9] With a homogeneous intensity distribution and a circular pupil of the impinging laser beam, the hologram function can be calculated analytically because of symmetrical conditions.

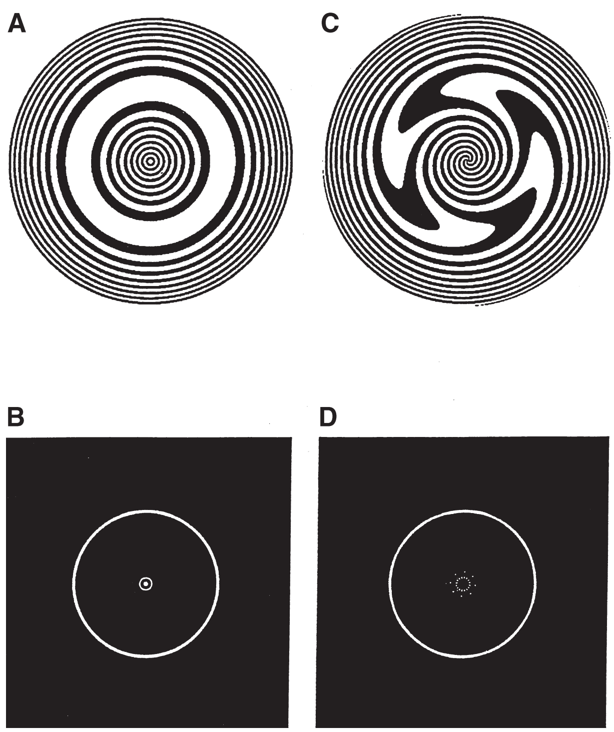

Figure 2. (**A**) Phase function of hologram with circular aperture focusing 2.94 μm Er:YAG laser beam with homogeneous intensity distribution into ring profile. Aperture diameter is 10 mm, inner radius of ring 7 mm, and outer radius 8 mm. Focal distance of the element is 100 mm. (**B**) Simulated reconstruction of ring with hologram specified in A. Poisson's spot on z-axis is consequence of constructive interference of -1 diffraction order. (**C**) Phase function of hologram specified in A with circular modulation of $j = 4$. This modulation leads to spiral-like structure in phase function. (**D**) Simulated reconstruction of ring with hologram specified in C. Poisson's spot on z-axis and circular region centered to z-axis with diameter 0.55 mm are without intensity. Intensity of reconstruction error is less compared to B.

Although the intensity of the reconstruction error of the hologram inside the ring is small with circular modulation of the phase, it has to be avoided in *donor* trephination. A centrally closed mask is necessary to eliminate ablation of the donor button centrally.

A small defocus during the trephination effects a trapezoidal asymmetry of the radial intensity distribution. This feature may be used for reducing the energy impinging on the mask causing a thermal load. For donor/recipient trephination, the working distance has to be reduced/enlarged slightly (ie, 1–3 mm) to reveal an asymmetric energy distribution with a decrease from the mask to the cornea. This effect will guarantee a maximum cut depth directly at the mask edge.

Stochastic laser energy fluctuations from pulse to pulse result in inhomogeneous trephination depths when a laser spot is guided along the trephination margin. This has been studied in detail for donor and recipient excimer laser trephination.[10] This disadvantage can be avoided with a ring focus, where the actual laser energy influences the ablation rate within the entire circumference.

For nonmechanical trephination with the ArF excimer laser, we developed a Fourier grating with a circular focus,[11] but the practical use was limited due to a fluence of the laser that was near the ablation threshold of the cornea (200 mJ/cm^2). The major difference between creating a ring focus for ultraviolet laser light and infrared light sources is that the structure of the grating for infrared lasers is larger by a factor of 10.

By means of computer-generated holograms, laser beams can be focused into almost arbitrary shapes. Elementary foci such as lines and rings can be realized with phase-only holograms. Moreover, it is possible to redistribute the energy along the desired focal curve. Thus, the optical element can be adapted to different intensity distributions of the incoming laser beam and different aperture shapes. The resulting total hologram function contains amplitude and phase information. Generally, it can be stated that the hologram phase is responsible for the directions of the refracted waves, whereas the amplitude is related to their relative intensity distribution. The resulting phase-only hologram allows on-axis reconstruction with high diffraction efficiency.

To avoid speckle noise resulting from discrepancies between phase values of adjacent object points within the optical reconstruction, a smooth phase function is elementary. This implies that there is a one-to-one correspondence between points in the hologram aperture and in the reconstruction plane. Different aperture shapes and intensity or phase distributions of the laser beam have to be taken into account during hologram design. An algorithm fulfilling these requirements has been reported in a generalized version by Golub et al.[5] This algorithm has to be modified for calculation of elements with homogeneous intensity distribution of the laser beam such as in the case of an Er:YAG laser.

In conclusion, with a computer-generated hologram it is possible to redistribute the energy along the desired focal ring. The hologram design can be adapted to the intensity distribution of the impinging laser beam with its characteristic aperture shape. With an azimuthal modulation of the phase, the energy density in the center (Poisson's spot) can be reduced beyond the ablation threshold of the cornea. A circular homogeneous trephination depth is possible, because the energy fluctuation from pulse to pulse does not locally affect the ablation process. A ring focus for the Er:YAG laser has the potential to render a manual beam control via micromanipulator superfluous.

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